3.15 Multiple Angle Equations

Often, equations involve expressions like $\sin 2x$, $\cos 3\alpha$, or $\tan\left(x/2\right)$, all of which involve multiples of the variable rather than a single variable. To solve these equations, we solve for the multiple variable just as we would solve for a single variable and then multiply or divide to get the single variable in the last step.

Example: Find all solutions in degrees to $\sin 2\alpha = \sqrt{3}/2$.

Example: Find all solutions to $\tan\left(4x\right) = 1$ in the interval $\left(0, \pi\right)$.

Example: Find all real number solutions to $\cos\left(x/2\right) = \sqrt{3}/2$.

Example: Find all solutions to $\csc\left(2x\right) = 2\sqrt{3}/3$ in the interval $\left(0°, 360°\right)$.

The Path of a Projectile

The distance *d* (in feet) traveled by a projectile fired from the ground with an angle of elevation θ is related to the initial velocity v_0 (in ft/sec) by the equation $v_0^2 \sin 2\theta = 32d$. If the projectile is fired from the origin into the first quadrant, then the *x*- and *y*-coordinates (in feet) of the projectile at time *t* (in seconds) are given by $x = v_0 t \cos\theta$ and $y = -16t^2 + v_0 t \sin\theta$.

Example: A catapult is placed 100 feet from the castle wall, which is 35 feet high. A soldier wants a burning bale of hay to clear the top of the wall and land 50 feet inside the castle wall. If the initial velocity of the bale is 70 ft/sec, then at what angle should the bale of hay be launched so that it will travel 150 feet and pass over the castle wall?